

# Testing CP-Violation in the Scalar Sector at Future $e^+e^-$ Colliders

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We propose a *model-independent* method to test CP-violation in the scalar sector through measuring the inclusive cross sections of  $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$  processes with the recoil mass technique, where  $h_1, h_2$  stand for the 125 GeV standard model (SM) like Higgs boson and a new lighter scalar respectively. This method effectively measures a quantity  $K$  proportional to the product of the three couplings of  $h_1ZZ, h_2ZZ, h_1h_2Z$  vertices. The value of  $K$  encodes a part of information about CP-violation in the scalar sector. We simulate the signal and backgrounds for the processes mentioned above with  $m_2 = 40\text{GeV}$  at the Circular Electron-Positron Collider (CEPC) with the integrated luminosity  $5\text{ab}^{-1}$ . We find that the discovery of both  $Zh_2$  and  $h_1h_2$  processes at  $5\sigma$  level will indicate an  $\mathcal{O}(10^{-2})$   $K$  value which can be measured to 16% precision. The method is applied to the weakly-coupled Lee model in which CP-violation can be tested either before or after utilizing a “ $p_T$  balance” cut (see section IIB for the definition). Lastly we point out that  $K \neq 0$  is a sufficient but not a necessary condition for the existence of CP-violation in the scalar sector, namely  $K = 0$  does not imply CP conservation in the scalar sector.

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## I. INTRODUCTION

CP-violation was first observed through  $K_L^0 \rightarrow \pi\pi$  decay in 1964 [1]. More CP-violation effects have been discovered in K- and B- meson sectors since then [2]. In 1973, Kobayashi and Maskawa propose [3] that if there exist three or more generations of fermions, one or more nontrivial phase(s) will be left in the quark mixing matrix, namely the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3, 4]. In the standard model (SM), only a single nontrivial phase is left which turns out to explain all the measured CP-violation effects successfully [2]. However, it is still necessary and attractive to study additional sources of CP-violation, which may help to understand the matter-antimatter asymmetry in the universe [2, 5].

In the SM, there is no CP-violation in the scalar sector. In models with additional scalars, extra CP-violation may be introduced in the scalar sector [6]. For example, in a minimal extension of SM [7], some kinds of two-Higgs-doublet models (2HDM) like Lee model [8] or Georgi model [9], and Weinberg model which contains three Higgs doublets [10], etc., there exists CP-violation in the scalar sector. In such models, a Higgs boson can be a CP-mixing state. As an example, two of the authors have studied the phenomenology of Lee model which contains spontaneous CP-violation in the scalar sector in detail [11–13]. These papers revealed the possible correlation between the lightness of Higgs boson and the smallness of CP-violation based on spontaneous CP-violation mechanism which provides another important motivation to study CP-violation further in the scalar sector.

In 2012, a SM-like Higgs boson was discovered by the ATLAS and CMS collaborations [14, 15] with its mass around 125 GeV [16]. Its spin and CP properties have also been studied through the final state distributions of  $h \rightarrow ZZ^* \rightarrow 4\ell$  decay process with the conclusion that a pure  $0^+$  state is favored and a pure  $0^-$  state is excluded at over  $3\sigma$  level [17–19]. However, a CP-mixing state is still allowed [17, 20] because the contribution from pseudoscalar component is loop induced and thus highly suppressed.

CP-violation beyond the SM may show several kinds of indirect effects<sup>1</sup>. For example, it may contribute to the electric dipole moments (EDM) of electron or neutron [22] which

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<sup>1</sup> Here “indirect” means these phenomena will show evidence for CP-violation, but we cannot extract the CP-violation vertex through these processes; while in the “direct” effects discussed below, we can obtain the CP-violation vertex through these measurements directly. Besides the effects discussed below, the Higgs cubic self coupling could also be modified [21] though the modification does not imply CP-violation.

are stringently constrained experimentally [23, 24]; it may contribute to meson mixing matrix element and thus a modification from SM prediction could occur [25]; or it may also contribute to the anomalous  $ZZZ$  coupling vertex [26, 27] which could lead to a nontrivial CP-sensitive asymmetry in  $e^+e^- \rightarrow ZZ$  process [28].

However, to study the exact sources of extra CP-violation, we need their direct effects. For example, a CP-mixing Higgs boson could couple to a fermion through the effective interaction

$$\mathcal{L}_{hf\bar{f}} = -h\bar{f}(g_S + ig_P\gamma^5)f, \quad (1)$$

where  $g_S$  and  $g_P$  may be of the same order. For  $f = \tau$ , it is possible to test CP-violation effects in  $h\tau^+\tau^-$  vertex at future  $pp$  or  $e^+e^-$  colliders [29–31] using the final state distribution of  $h \rightarrow \tau^+\tau^- \rightarrow \nu\bar{\nu} + X$  decay process. Similarly, for  $f = t$ , the top polarization asymmetry in  $e^+e^- \rightarrow t\bar{t}h$  process is useful to test CP-violation effects in  $ht\bar{t}$  vertex [32].

In this paper, we will focus on the scalar sector itself and propose a *model-independent* method to test CP-violation effects in the scalar sector through the interaction between scalars and massive gauge bosons. The paper is organized as follows. In section II we describe our method and perform a simulation study at the CEPC. In section III we apply this method to the weakly-coupled Lee model. And in section IV we give our conclusions and discussions.

## II. MODEL-INDEPENDENT METHOD TO TEST CP-VIOLATION IN THE SCALAR SECTOR AT FUTURE $e^+e^-$ COLLIDERS

If more than one neutral scalars are discovered in the future, the tree level interaction between neutral scalars and massive gauge bosons could be written as

$$\mathcal{L}_{\text{tree}} = \sum_i c_i h_i v \left( \frac{g^2}{2} W^{+\mu} W_{\mu}^{-} + \frac{g^2}{4c_W^2} Z^{\mu} Z_{\mu} \right) + \sum_{i < j} \frac{c_{ij}g}{2c_W} Z_{\mu} (h_i \partial^{\mu} h_j - h_j \partial^{\mu} h_i). \quad (2)$$

Here  $g$  is the  $SU(2)_L$  coupling constant,  $c_W$  denotes the cosine of electro-weak angle  $\theta_W$ <sup>2</sup>,  $v$  is the vacuum expected value for SM scalar field, and  $h_i$  represents the  $i$ th scalar. For the first two terms, a nonzero tree-level  $h_i VV$  vertex requires that  $h_i$  must contain CP-even component; while for the last term, a nonzero tree-level  $h_i h_j Z$  vertex requires that  $h_i$  and

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<sup>2</sup> In this paper, we denote  $s_{\alpha} \equiv \sin \alpha$ ,  $c_{\alpha} \equiv \cos \alpha$ , and  $t_{\alpha} \equiv \tan \alpha$  for any angle  $\alpha$ .

$h_j$  must contain components with different CP-properties. If CP is a good symmetry, there must be some terms vanishing in (2); on the other hand, if all  $c_i$  and  $c_{ij}$  are nonzero, there must be CP-violation in the scalar sector.

### A. Method for the Minimal Case

For the minimal case, two neutral scalars with non-degenerate masses are required to be discovered. CP-violation can be confirmed with  $c_1$ ,  $c_2$ , and  $c_{12}$  all measured to be nonzero. It is natural to define

$$K \equiv c_1 c_2 c_{12} \quad (3)$$

which is a useful quantity to measure the CP-violation effect since  $K \neq 0$  is a sufficient condition for the existence of CP-violation in the scalar sector<sup>3</sup>. As an example, in 2HDMs, there are three neutral Higgs bosons. We can use this idea to search for direct CP-violation effect once two of them are discovered. A straightforward calculation shows  $c_{12} = c_3$ , and  $K$  is just the product for all  $c_i$  in 2HDM. That is an important quantity to measure CP-violation in the scalar sector [26–28, 33].

At the LHC, the 125 GeV Higgs boson  $h_1$  has already been discovered and the direct  $h_1 VV$  vertices have been confirmed [17, 34]. If another Higgs boson  $h_2$  is discovered and it has tree level<sup>4</sup> decay channels  $h_2 \rightarrow WW, ZZ, Zh_1$ , it would strongly suggest CP-violation in the scalar sector which has already been discussed in [11, 13, 36]. However, the  $\sigma \cdot \text{Br}$  measurements at LHC depend on not only  $c_{1,2}$  and  $c_{12}$ , but also a lot of other parameters which would affect on the production cross section or branching ratios. Thus it is difficult to extract or constrain the value of  $K$  from these measurements without model-dependent assumptions.

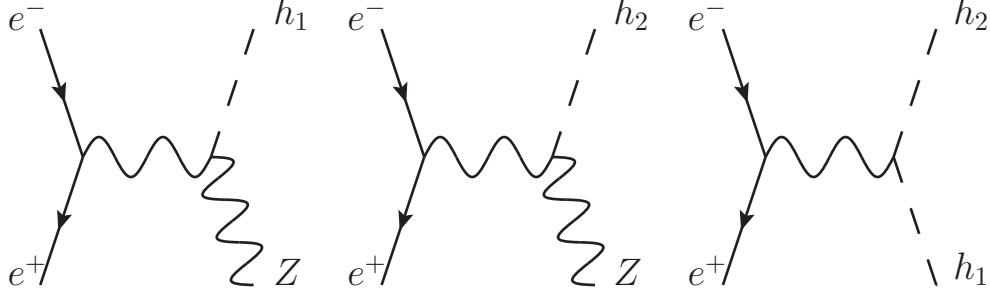
At future  $e^+e^-$  colliders, we can use three associated production processes,  $e^+e^- \rightarrow Z^* \rightarrow Zh_1, Zh_2, h_1h_2$ , to search for CP-violation in the scalar sector. The Feynman diagrams are shown in Figure 1. The cross sections at tree-level are given as [37, 38]

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<sup>3</sup> One should aware that  $K \neq 0$  is not a necessary condition for the existence of CP-violation in the scalar sector which means in some models, there may be CP-violation in the scalar sector with  $K = 0$ , see the discussions in the last section.

<sup>4</sup> In some special models, for example, the loop-philic model [35], a loop-induced decay channel can also have a large branching ratio even it is weakly-coupled.

FIG. 1: Feynman diagrams for associated production processes  $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$ .



$$\sigma_{Zh_i} = \frac{\pi\alpha^2 s \cdot c_i^2}{96(s - m_Z^2)^2} \left( \frac{8s_W^4 - 4s_W^2 + 1}{s_W^4 c_W^4} \right) \left( f^3 \left( \frac{m_i^2}{s}, \frac{m_Z^2}{s} \right) + \frac{12m_Z^2}{s} f \left( \frac{m_i^2}{s}, \frac{m_Z^2}{s} \right) \right); \quad (4)$$

$$\sigma_{h_i h_j} = \frac{\pi\alpha^2 s \cdot c_{ij}^2}{96(s - m_Z^2)^2} \left( \frac{8s_W^4 - 4s_W^2 + 1}{s_W^4 c_W^4} \right) f^3 \left( \frac{m_i^2}{s}, \frac{m_j^2}{s} \right). \quad (5)$$

Here  $s$  is the square of total energy in the center-of-mass frame,  $s(c)_W$  denotes the (co)sine of electro-weak angle  $\theta_W$ , and the function

$$f(x, y) \equiv \sqrt{1 + x^2 + y^2 - 2x - 2y - 2xy}. \quad (6)$$

The cross sections are sensitive to  $c_i$  or  $c_{ij}$ , but besides these, they don't depend on more details of the model.

The recoil mass technique [39–41] would be very effective for precision measurements on these inclusive cross sections. For  $e^+e^- \rightarrow Z(f\bar{f})h_i$  process, the recoil mass is defined as [39, 40]

$$m_{\text{rec}} \equiv \sqrt{s + m_{f\bar{f}}^2 - 2\sqrt{s}(E_f + E_{\bar{f}})} \quad (7)$$

whose distribution would show a narrow peak around  $m_i$  where  $m_{f\bar{f}}^2 = m_Z^2$  is the invariant mass of the fermion pair. With this method, the sensitivity to  $Zh_1$  inclusive cross section would reach better than 1% at future Higgs factories [41–43] with  $\sqrt{s} = 250\text{GeV}$  and  $\mathcal{O}(\text{ab}^{-1})$  luminosity. The result doesn't depend on the decay channels of Higgs boson which means this is a model-independent technique to measure  $h_i ZZ$  couplings  $c_i$ . Generalizing this technique to  $e^+e^- \rightarrow h_1(b\bar{b})h_2$  process, with  $h_1$  the 125 GeV Higgs boson and  $m_{b\bar{b}}^2 = m_1^2$ , the distribution of  $m_{\text{rec}}$  would show a narrow peak around  $m_2$  and thus we can measure the  $e^+e^- \rightarrow h_1h_2$  inclusive cross section to extract the  $h_1h_2Z$  coupling  $c_{12}$  in a model-independent way<sup>5</sup>. Thus through measuring the three inclusive associated production cross

<sup>5</sup> In order to measure  $\sigma_{h_1h_2}$  using this method,  $\text{Br}(h_1 \rightarrow b\bar{b})$  is needed as a model-dependent quantity, which can be accurately measured through  $e^+e^- \rightarrow Zh_1$  process.

sections, we can extract all the three couplings  $c_1, c_2, c_{12}$  and subsequently obtain  $K$  in a model-independent way.

## B. Model-Independent Simulation Study

Here we perform a simulation study of the signal and backgrounds for the case  $m_2 = 40\text{GeV}$  at Circular Electron-Positron Collider (CEPC) [41] which would be a  $e^+e^-$  collider with  $\sqrt{s} = 250\text{GeV}$ <sup>6</sup>. Such a light scalar can occur in many models, such as 2HDMs [6, 12, 13, 45, 46].

Assuming  $h_1$  is SM-like,  $c_1 \sim 1$  which is consistent with the recent 125 GeV Higgs measurements [47]. In the following we focus on the inclusive measurements on  $Zh_2$  and  $h_1h_2$  associated production processes. The strictest direct constraints on  $c_2$  and  $c_{12}$  came from LEP results [48, 49] which give

$$|c_2| < 0.18, \quad |c_{12}| < 0.54 \quad (8)$$

for  $m_2 = 40\text{GeV}$  at 95% C.L. assuming all scalars decay only to  $b\bar{b}$  final states.

In our simulation analysis, we use WHIZARD-2.3.1 [50] to generate signal and background events with initial state radiation (ISR) and beamstrahlung effects. For beamstrahlung effects, we use the built-in spectra CIRCE2 for the CEPC project [51]. For both processes, we adopt the recoil mass method in which we do not reconstruct  $h_2$  directly using its decay final states thus the results do not depend on the properties of  $h_2$ .

For  $Zh_2$  process, we choose the  $Z \rightarrow \mu^+\mu^-$  decay channel. The corresponding backgrounds are  $e^+e^- \rightarrow \mu^+\mu^-X$  where  $X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, b\bar{b}, \nu\bar{\nu}$ , or  $\gamma\gamma$  [41, 52–54]. We impose the basic cuts as [41, 52]

$$\begin{aligned} |\cos\theta_{\mu^\pm}| &< 0.98, \quad m_{\mu^+\mu^-} > 15\text{GeV}, \quad m_{\text{rec}} > 15\text{GeV}, \\ |\cos\theta_{e^\pm, \gamma}| &< 0.995, \quad E_\gamma > 0.1\text{GeV}, \quad \Delta R_{ij} > 0.4. \end{aligned} \quad (9)$$

where  $m_{\text{rec}}$  is defined in (7) with  $f = \mu$  and  $\Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$  with  $i$  and  $j$  running over all partons in the final state<sup>7</sup>. The transverse momentum of muon is smeared

<sup>6</sup> If the extra scalar is a heavier one, we can utilize this method at  $e^+e^-$  colliders with larger  $\sqrt{s}$ , like ILC [44].

<sup>7</sup> The cuts in the second line are useful to avoid the infrared and collinear divergences in background processes.

by a Gaussian distribution with the standard deviation of [41]

$$\sigma_{1/p_T} = 2 \times 10^{-5} \oplus 1 \times 10^{-3}/(p_T \sin \theta) [\text{GeV}^{-1}]. \quad (10)$$

We impose the selection cuts as

$$\begin{aligned} |\cos \theta_{\mu^\pm}| < 0.8, \quad p_T(\mu^+ \mu^-) > 35 \text{GeV}, \quad |m_{\mu^+ \mu^-} - m_Z| < 10 \text{GeV}, \\ \text{and} \quad 30 \text{GeV} < m_{\text{rec}} < 60 \text{GeV}. \end{aligned} \quad (11)$$

After all the selection cuts, we have

$$\sigma_{\text{sig}} = c_2^2 \times 7.438 \text{fb}, \quad \sigma_{\text{bkg}} = 5.916 \text{fb}, \quad (12)$$

in which  $\sigma_{\mu^+ \mu^- \gamma \gamma} = 4.659 \text{fb}$  is the dominant background process. Moreover, we can take advantage of the “ $p_T$  balance” cut [54, 55] to suppress the  $\mu^+ \mu^- \gamma \gamma$  background further. The observable  $p_{T,\text{bal}}$  is defined as

$$p_{T,\text{bal}} \equiv p_T(\mu^+ \mu^-) - p_T(\gamma) \quad (13)$$

where  $p_T(\gamma)$  is the transverse momentum of the most energetic photon tagged<sup>8</sup>. If we choose the cut  $p_{T,\text{bal}} > 20 \text{GeV}$  as [55], we have

$$\sigma'_{\mu^+ \mu^- \gamma \gamma} = 0.211 \text{fb} \quad \text{thus} \quad \sigma'_{\text{bkg}} = 1.468 \text{fb} \quad (14)$$

with cross sections of other processes unchanged. Using these results, we summarize the  $3\sigma$ ,  $5\sigma$  discovery potential and expected 95% C.L. upper limit (corresponding to  $1.64\sigma$ ) on  $|c_2|$  with  $5 \text{ab}^{-1}$  luminosity at CEPC before and after “ $p_T$  balance” cut separately in Table I.

TABLE I: Expected 95% C.L. upper limit,  $3\sigma$ , and  $5\sigma$  discovery potential for  $|c_2|$  with  $5 \text{ab}^{-1}$  luminosity at CEPC.

	95% C.L. limit	$3\sigma$ discovery	$5\sigma$ discovery
before “ $p_T$ balance” cut	$< 0.087$	$> 0.118$	$> 0.152$
after “ $p_T$ balance” cut	$< 0.061$	$> 0.083$	$> 0.107$

<sup>8</sup> With this method, we must tag at least one photon which breaks the inclusiveness of the measurement. But for most cases, we can assume  $\text{Br}(h \rightarrow \gamma\gamma) \ll 1$  so that tagging a photon would make only a little difference on the measurement.

For  $h_1 h_2$  process, we use the  $h_1 \rightarrow b\bar{b}$  decay channel. The backgrounds include  $e^+e^- \rightarrow b\bar{b}X$  and  $e^+e^- \rightarrow Zh_1(b\bar{b})$  where  $X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, b\bar{b}, \nu\bar{\nu}, \gamma\gamma, g\gamma$ , and  $gg$ <sup>9</sup>. We impose the basic cuts as

$$m_{b\bar{b}} > 15\text{GeV}, \quad m_{\text{rec}} > 15\text{GeV},$$

$$|\cos\theta_{e^\pm, \gamma}| < 0.995, \quad E_\gamma > 0.1\text{GeV}, \quad E_g > 1\text{GeV}, \quad \Delta R_{ij} > 0.4 \quad (15)$$

where  $m_{\text{rec}}$  is defined in (7) with  $f = b$  and  $\Delta R_{ij}$  run over all partons in the final state<sup>10</sup>. The jet energy is smeared by a Gaussian distribution with the standard deviation of [41]

$$\frac{\sigma_E}{E} = \frac{0.3}{\sqrt{E(\text{GeV})}} \quad (16)$$

for the jet energy less than 100GeV. The  $b$ -tagging efficiency and  $c$ -faking rate are [41]

$$\epsilon_b = 0.9, \quad P_{c \rightarrow b} = 0.1 \quad (17)$$

separately. In an event, at least two  $b$  jets should be tagged. The candidates of  $b$  jets from  $h_1$  decays are selected with the minimal  $|m_{b\bar{b}} - m_1|$  and then sorted by the transverse momenta. The leading and sub-leading  $p_T$  of the selected  $b$  jet pairs are denoted as  $p_T(b)$  and  $p_T^{\text{sub}}(b)$ .

We impose the selection cuts as

$$70\text{GeV} < p_T(b\bar{b}) < 100\text{GeV}, \quad 70\text{GeV} < p_T(b) < 110\text{GeV}, \quad 30\text{GeV} < p_T^{\text{sub}}(b) < 70\text{GeV},$$

$$|m_{b\bar{b}} - m_1| < 25\text{GeV}, \quad \text{and} \quad 20\text{GeV} < m_{\text{rec}} < 70\text{GeV}. \quad (18)$$

After these selection cuts, we have

$$\sigma_{\text{sig}} = \frac{c_{12}^2 \text{Br}(h_1 \rightarrow b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b})} \times 12.5\text{fb}, \quad \sigma_{\text{bkg}} = \left( 20.54 + 0.577 \left( \frac{\text{Br}(h_1 \rightarrow b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b})} \right) \right) \text{fb} \quad (19)$$

where  $\text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b}) = 0.5824$  [56] for  $m_1 = 125\text{GeV}$ . The dominant background is  $b\bar{b}gg$  production with its cross section  $\sigma_{b\bar{b}gg} = 13.2\text{fb}$ . The backgrounds with photon have the cross section  $\sigma_{b\bar{b}g\gamma + b\bar{b}\gamma\gamma} = 4.981\text{fb}$  which can be suppressed to  $\sigma'_{b\bar{b}g\gamma + b\bar{b}\gamma\gamma} = 1.107\text{fb}$  using the

<sup>9</sup> We also considered other background processes like  $e^+e^- \rightarrow b\bar{b}h_{1,2}$  and  $e^+e^- \rightarrow Z(b\bar{b})h_2$ . However, numerically they are all negligible, thus we don't list them here

<sup>10</sup> The cuts in the second line are useful to avoid the infrared and collinear divergences in background processes as discussed above.



“ $p_T$  balance” cut discussed above. The “ $p_T$  balance” cut does not affect on signal and other background processes thus the total background can be reduced to

$$\sigma'_{\text{bkg}} = \left( 16.66 + 0.577 \left( \frac{\text{Br}(h_1 \rightarrow b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b})} \right) \right) \text{fb}. \quad (20)$$

As a benchmark point, take  $\text{Br}(h_1 \rightarrow b\bar{b}) = \text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b})$ . We use the results above to summarize the  $3\sigma$ ,  $5\sigma$  discovery potential and expected 95% C.L. upper limit on  $|c_{12}|$  with  $5\text{ab}^{-1}$  luminosity at CEPC before and after “ $p_T$  balance” cut separately in Table II.

TABLE II: Expected 95% C.L. upper limit,  $3\sigma$ , and  $5\sigma$  discovery potential for  $|c_{12}|$  with  $5\text{ab}^{-1}$  luminosity at CEPC.

	95% C.L. limit	$3\sigma$ discovery	$5\sigma$ discovery
before “ $p_T$ balance” cut	$< 0.092$	$> 0.125$	$> 0.161$
after “ $p_T$ balance” cut	$< 0.088$	$> 0.119$	$> 0.153$

### III. IMPLICATION FOR WEAKLY-COUPLED LEE MODEL

In this paper, we choose weakly-coupled Lee model [12, 13] which naturally contains a light scalar in small CP-violation limit as a benchmark model to study the implications of our simulation results.

Lee model was proposed by Lee in 1973 [8] as a 2HDM which is CP-conserved at Lagrangian level but the CP-violation comes from the vacuum. The scalar potential can be written as

$$\begin{aligned} V(\phi_1, \phi_2) = & \mu_1^2 R_{11} + \mu_2^2 R_{22} + \lambda_1 R_{11}^2 + \lambda_2 R_{11} R_{12} \\ & + \lambda_3 R_{11} R_{22} + \lambda_4 R_{12}^2 + \lambda_5 R_{12} R_{22} + \lambda_6 R_{22}^2 + \lambda_7 I_{12}^2 \end{aligned} \quad (21)$$

where  $R(I)_{ij}$  is the real (imaginary) part of  $\phi_i^\dagger \phi_j$ . Both  $\phi_i$  are scalar doublets which can be written as  $\phi_1 = (\phi_1^+, (v_1 + R_1 + iI_1)/\sqrt{2})^T$  and  $\phi_2 = (\phi_2^+, (v_2 \exp(i\xi) + R_2 + iI_2)/\sqrt{2})^T$ . Here  $I_{1,2}$  and  $R_{1,2}$  are scalar degrees of freedom and  $v = \sqrt{v_1^2 + v_2^2} = 246\text{GeV}$ . According to the vacuum stability condition, if

$$|\lambda_2 v_1^2 + \lambda_5 v_2^2| < 2|\lambda_4 - \lambda_7|v_1 v_2, \quad (22)$$

a nontrivial phase difference  $\xi$  between the vacuum expected values (VEV) of the two Higgs doublets would arise thus CP symmetry is spontaneously broken. As a consequence all the three neutral Higgs bosons must be CP-mixing states.

Defining  $t_\beta \equiv v_2/v_1$ , for weakly-coupled scalar sector ( $\lambda_i \lesssim \mathcal{O}(1)$ ), in the limit of small  $t_\beta s_\xi$ , a new light scalar is predicted with the mass  $m_2 \sim \mathcal{O}(vt_\beta s_\xi)$  [11–13]. We treat it as the 40GeV new scalar. Its couplings to massive vector bosons are also suppressed by  $c_2 \sim \mathcal{O}(t_\beta s_\xi) \sim \mathcal{O}(0.1)$ . If the heavy Higgs boson has its mass  $m_3 \sim \mathcal{O}(v)$ , there is also additional constraint on  $c_{12}$  from LHC results [47]. If  $200\text{GeV} < m_3 < 300\text{GeV}$ ,  $c_{12} \equiv c_3 \lesssim (0.3 - 0.4)$  [12, 13, 57] which is stricter than the LEP result. In this scenario, the 125GeV Higgs boson  $h_1$  has SM-like couplings. The  $h_1 \rightarrow 2h_2$  decay channel measurements impose a strict constraint on  $h_1 h_2 h_2$  coupling to  $\mathcal{O}(10^{-2})$  [12, 58], but this measurement does not give tighter constraints on the  $c_1$ ,  $c_2$  and  $c_{12}$  couplings. The study in [12] and its update results in [57] showed this scenario is still viable facing all experimental constraints.

The results we obtained above showed we can set stricter constraint or discovery potential on this scenario. For  $h_1 h_2$  production channel, we use  $\text{Br}(h_1 \rightarrow b\bar{b}) = \text{Br}_{\text{SM}}(h_1 \rightarrow b\bar{b})$  as a benchmark point. Assuming all  $c_{1,2,12} > 0$ , we have

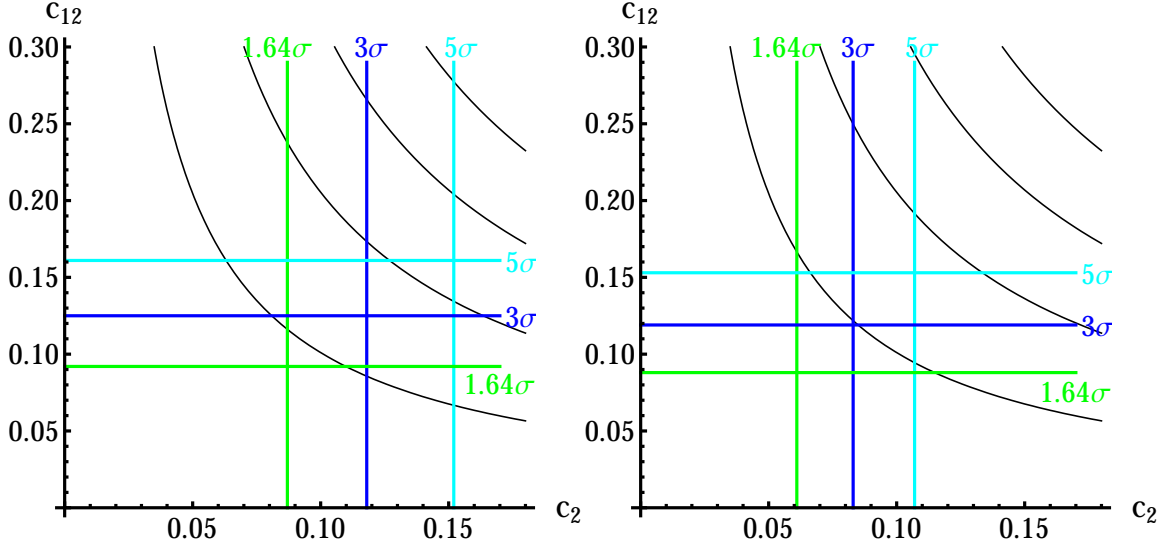
$$K = c_2 c_{12} \sqrt{1 - c_2^2 - c_{12}^2}. \quad (23)$$

In Figure 2 we show the expected limit or significance for different  $(c_2, c_{12})$  points before (see the left figure) or after (see the right figure) “ $p_T$  balance” cut discussed above. The four curves are  $K = 0.1, 0.2, 0.3, 0.4$  separately from left to right.

If there is no hint for either processes before “ $p_T$  balance” cut, it is expected to set an upper limit  $K < 7.9 \times 10^{-3}$ ; while the upper limit is expected to be  $K < 5.3 \times 10^{-3}$  after “ $p_T$  balance” cut. If both processes are discovered at over  $3(5)\sigma$  level before “ $p_T$  balance” cut, we have  $K > 1.5(2.4) \times 10^{-2}$ ; while the number should be  $1.0(1.6) \times 10^{-2}$  after “ $p_T$  balance” cut. In this case, we can confirm CP-violation in the scalar sector and measure  $K$  to the accuracy  $\delta K/K \lesssim 24(16)\%$ . For the case with the largest  $K$ , both couplings are set to the recent allowed upper limit,  $c_2 = 0.18$  and  $c_{12} = 0.3$ , we will have  $K = 5.4 \times 10^{-2}$  and  $\delta K/K = 7.9(4.7)\%$  before (after) “ $p_T$  balance” cut.

In the discussions above, we just use the inclusive measurements to determine the couplings and hence  $K$  in a model-independent way. For the discovery potential of a specific model, it would be better to use exclusive decay channels such as  $h_2 \rightarrow b\bar{b}$  which is expected

FIG. 2: Expected limit or significance for different  $(c_2, c_{12})$  points. The left figure is for the results before “ $p_T$  balance” cut while the right figure is for the results after “ $p_T$  balance” cut thus it is a “quasi-inclusive” result. The four curves are for  $K = 0.1, 0.2, 0.3, 0.4$  from left to right. We denote the boundary of  $1.64\sigma$ ,  $3\sigma$ , and  $5\sigma$  significance with green, blue, and cyan lines respectively.



to be dominant in most models. The sensitivity would also increase if we combine the results from more decay channels of  $Z$  and  $h_1$ .

#### IV. CONCLUSIONS AND DISCUSSIONS

Once two scalars are discovered, we can test the CP-violation in the scalar sector through searching for nonzero tree-level  $h_1ZZ, h_2ZZ, h_1h_2Z$  vertices according to the CP-properties analysis. Based on this idea, we proposed a model-independent method to confirm CP-violation in the scalar sector at future  $e^+e^-$  colliders through measuring the inclusive  $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$  cross sections with recoil mass technique. We can use a quantity  $K = c_1c_2c_{12}$  which is defined in (3) to measure CP-violation in the scalar sector.

We have performed simulation studies for  $m_2 = 40\text{GeV}$  at CEPC assuming the 125 GeV Higgs boson  $h_1$  is SM-like and the results are shown in Table I and Table II. We have adopted the recoil mass technique to ensure the measurements are inclusive<sup>11</sup>. The  $5\sigma$  discovery limit for both  $c_2$  and  $c_{12}$  are below the recent 95% C.L. upper limits. For  $Zh_2$  associated

<sup>11</sup> After “ $p_T$  balance” cut, it is quasi-model-independent as discussed above.

production, the “ $p_T$  balance” cut is efficient to drop the photon background but it also lose the inclusiveness a little. We choose the weakly-coupled Lee model which contains CP-violation and allows an extra light scalar as a benchmark model<sup>12</sup>. In the weakly coupled Lee model, both processes,  $Zh_2$  and  $h_1h_2$ , are possible to be discovered at  $5\sigma$  level before or after “ $p_T$  balance” cut. If both processes are discovered at  $3(5)\sigma$  level,  $K$  must reach  $\mathcal{O}(10^{-2})$  and the sensitivity of  $\delta K/K$  measurement can reach 24(16)%. This method is also applicable for other  $e^+e^-$  colliders if all the three processes can be discovered. For example, if the extra scalar is heavier, we can use this method at a  $e^+e^-$  collider with larger  $\sqrt{s}$ , such as ILC.

We should note that  $K \neq 0$  is a sufficient but not necessary condition for the existence of CP-violation in the scalar sector. Precisely speaking, we can use this method to confirm the existence of CP-violation in the scalar sector according to the nonzero  $K$ , but can’t constrain or exclude the CP-violation in the scalar sector if  $K$  is unmeasurable small. For example, in the minimal extension of SM mentioned above [7], there is only an additional complex singlet in the extension of the scalar sector. For some parameter choices, the three scalars would become CP-mixing states, but there are still no tree-level  $h_ih_jZ$  vertices thus the measurement on  $e^+e^- \rightarrow h_ih_j$  cannot give a positive result.

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<sup>12</sup> For most perturbative models, this method is useful to extract tree level information instead of loop level since loop-induced processes have small enough cross sections.

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